

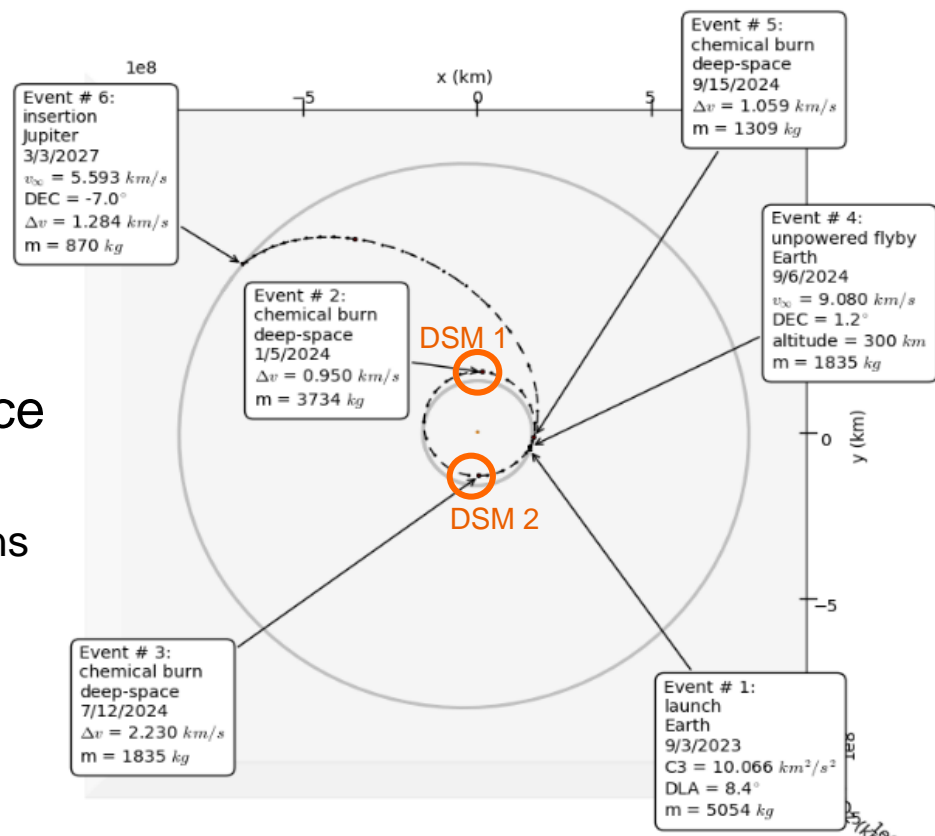
GLOBAL OPTIMIZATION OF n -MANEUVER, HIGH-THRUST TRAJECTORIES USING DIRECT MULTIPLE SHOOTING

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- Optimal number of maneuvers not known a priori
 - Deep-space maneuvers (DSMs) frequently improve performance
 - Two (or more) DSMs can be optimal for tightly/uniquely constrained trajectory legs
- Missions interested in best performance possible, i.e., the global optimum
 - Can enable cost- or time-constrained missions
 - Often interested in more than 1 objective, e.g., max. delivered mass & min. TOF
- Problem challenges:
 - Grid searches can be intractable for any number & variety of DSMs
 - Optimization requires initial guess
 - Maneuvers and gravity assists can create highly sensitive optimization problems

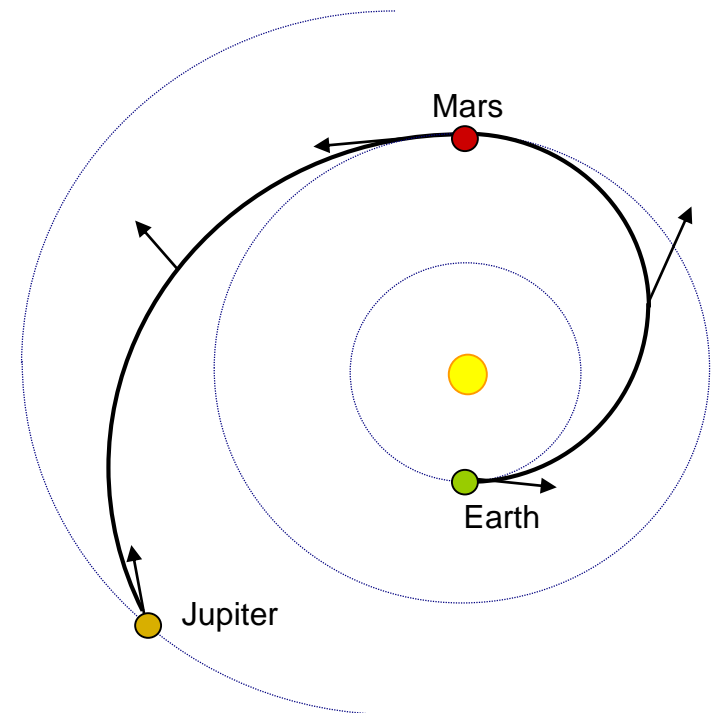
Optimal EEJ trajectory w/ 2 DSMs on 1 leg



Globally optimize chemical propulsion based trajectories with an arbitrary number of maneuvers & gravity assists

Method should be:

- Automated
- No requirement of a user-defined initial guess
- Able to search broad design space
- Efficient (medium-fidelity is appropriate)
- Capable of handling multiple objectives



Prior Approaches to Global Search



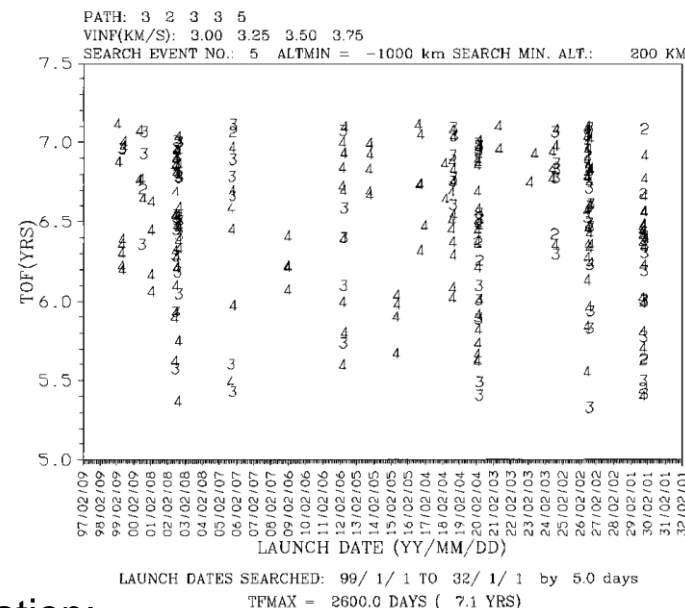
- Grid search

- Lambert scans over range of event dates & flyby bodies
- Strategies developed to include one maneuver per leg
 - Patel and Longuski → STOUR (Purdue, JPL)
 - Lantukh and Russell (UT-Austin)
 - Maneuvers limited to specific type v-infinity leveraging or broken plane maneuver

- Stochastic search

- Strategically sample design space
- Typically use a direct, Lambert-based trajectory formulation:
 - Vary departure date, time to DSM, & DSM components
 - Propagate forward to DSM point
 - Solve Lambert problem to subsequent body, repeat for all legs
 - Guaranteed feasibility for unconstrained problems
- Not limited in maneuver type, but problems are frequently very sensitive to initial guess
- Most approaches not capable of more than 1 DSM
- Global-local hybrid scheme can improve efficacy

Example STOUR plot (TOF vs. LD)
Petroopoulos et al., "Trajectories to Jupiter..." JSR, 2000



MGAnDSMs Trajectory Transcription

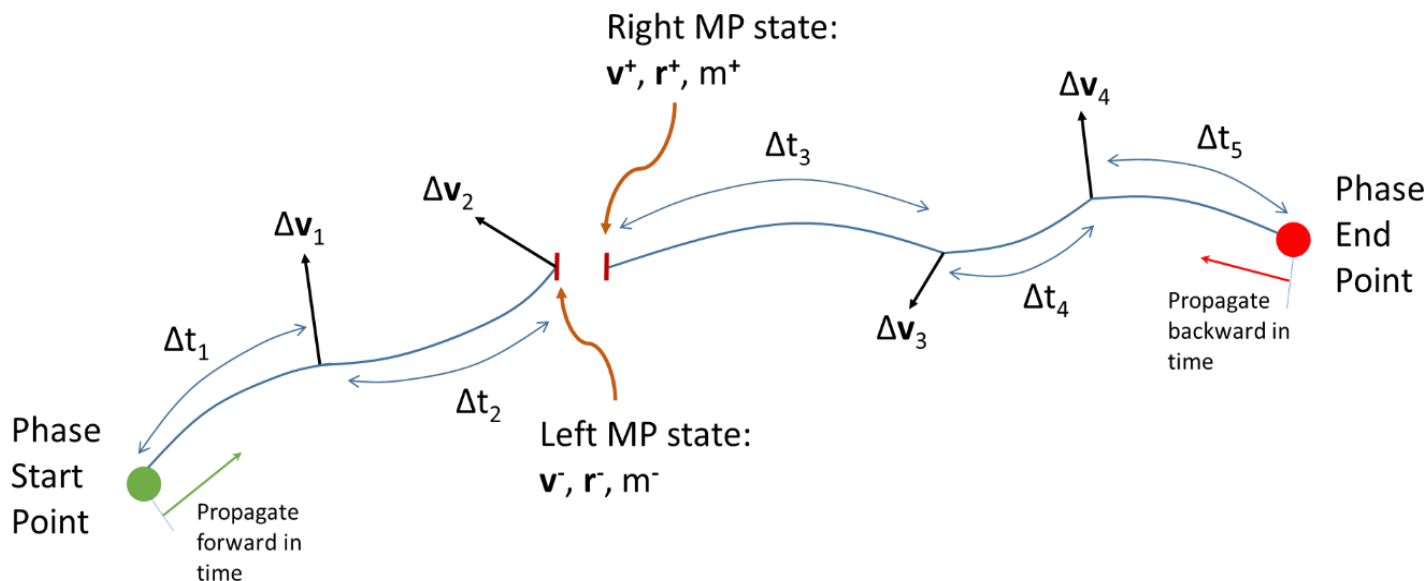


Multiple gravity assist with n deep space maneuvers using shooting scheme s

→ MGAnDSMs

- Aim for robustness & efficiency with an arbitrary number of DSMs with a direct formulation
- Nonlinear programming problem
- Employs forward/backward shooting similar to Byrnes & Bright (CATO)
- Nominally Kepler propagation between maneuvers
- Analytic match point (MP) constraints
- User specifies n , optimizer reduces 1 or more DSM magnitude to zero if $< n$ optimal

$n = 4$ maneuvers



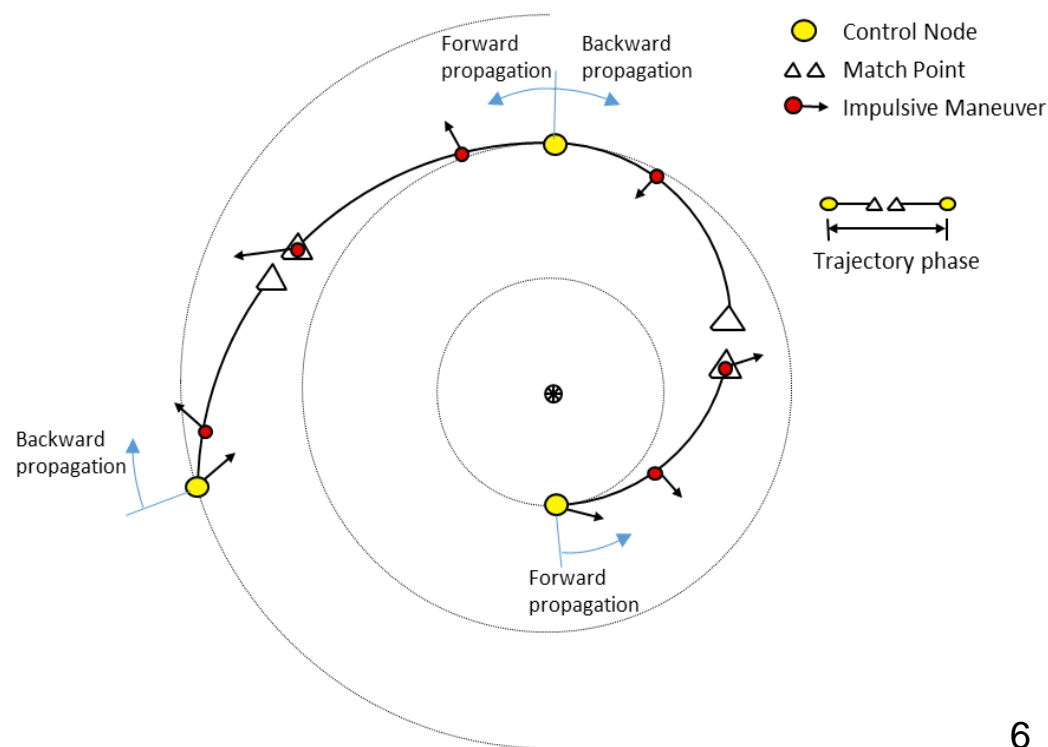
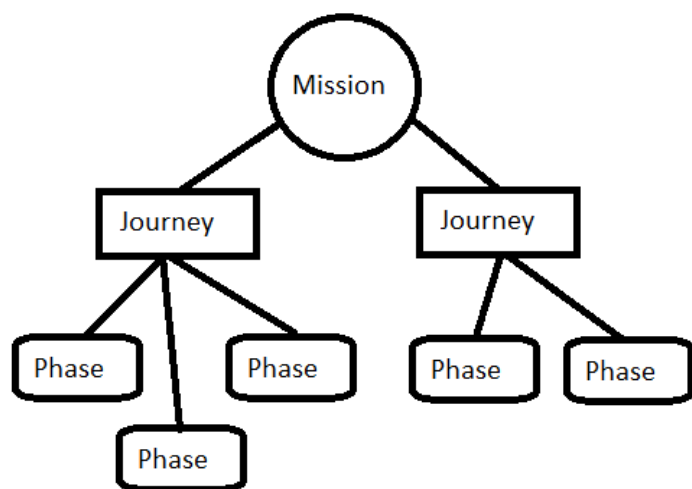
Core Optimization Variables

Δt_{phase}
Δt_1
$\Delta t_2, \dots, \Delta t_n$
Δt_{n+1}
$\Delta \mathbf{v}_1, \Delta \mathbf{v}_2, \dots, \Delta \mathbf{v}_n$
$\mathbf{v}_{\text{initial}}$
$\mathbf{v}_{\text{final}}$

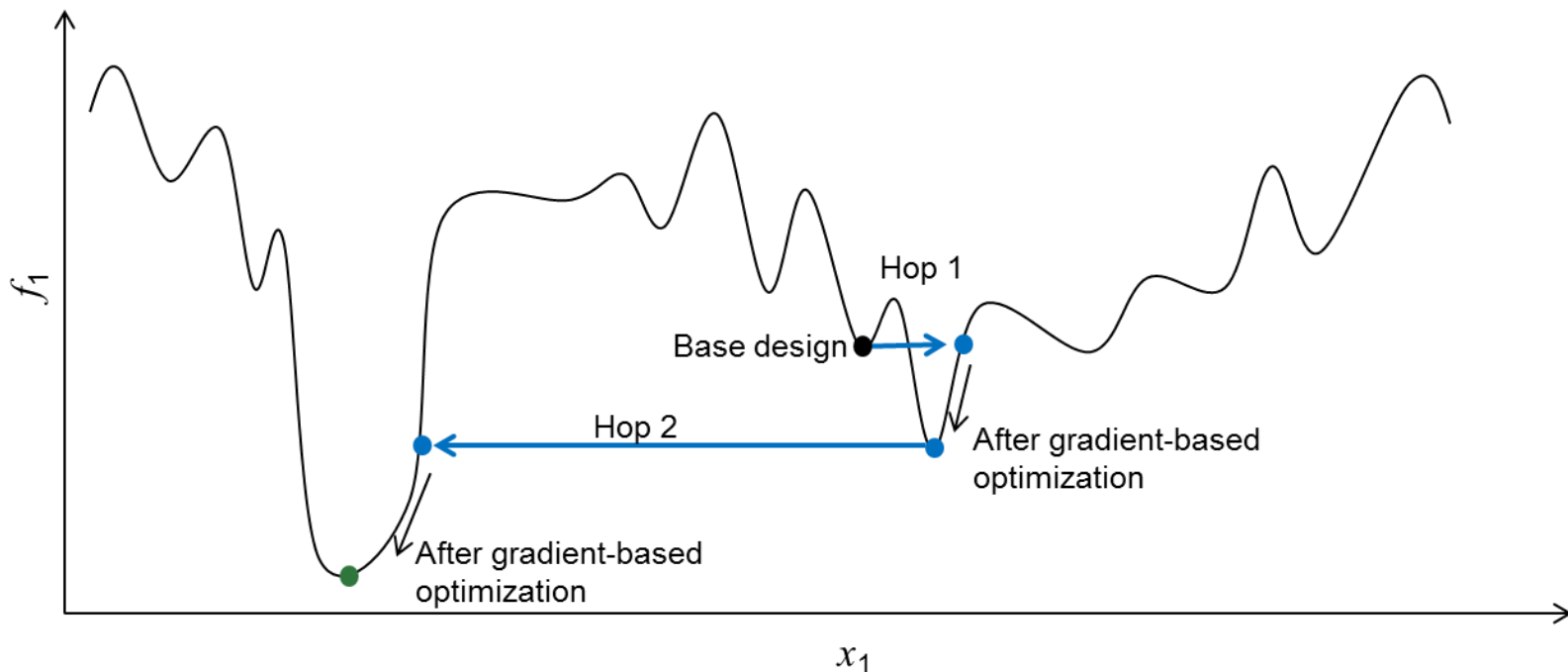
Match point

continuity constraint: $\mathbf{c}_{mp} = [r_x^+ - r_x^-, r_y^+ - r_y^-, r_z^+ - r_z^-, v_x^+ - v_x^-, v_y^+ - v_y^-, v_z^+ - v_z^-, m^+ - m^-]^T = \boldsymbol{\varepsilon}^T$

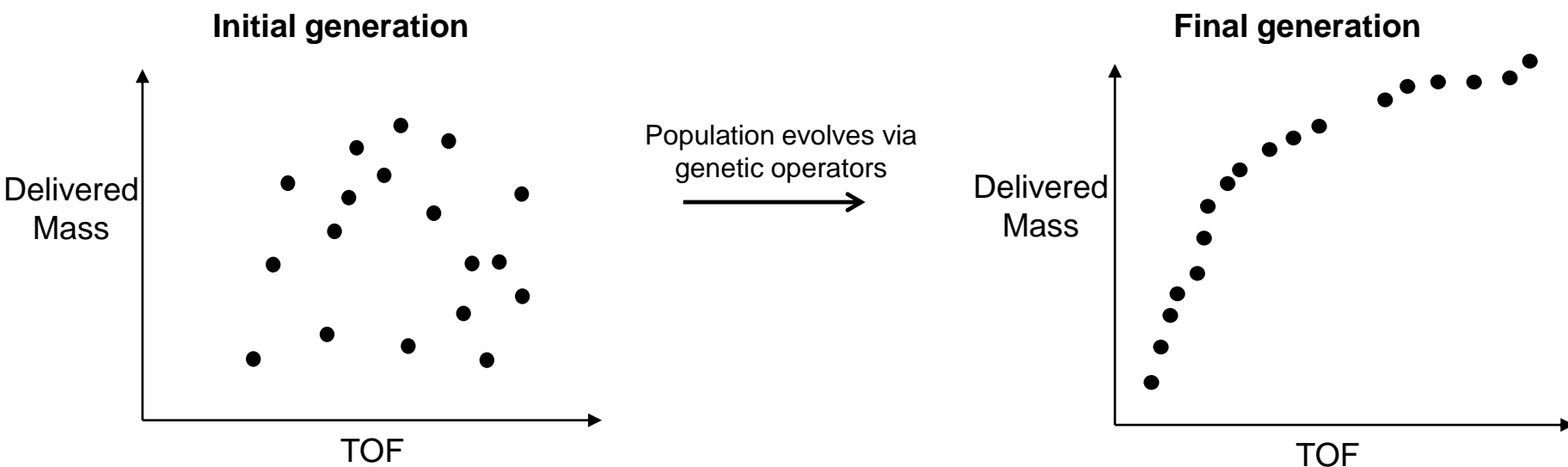
- Multiple gravity assists accommodated in mission structure
- Allows for variation of flyby bodies at control nodes of transcription
 - Journeys start & end a user-required bodies/states
 - Journeys can be composed of multiple phases with variable flyby bodies to improve performance (“null gene” approach)
- Zero sphere of influence patched conics
 - Flyby constraints ensure physically realizable gravity assist
- Flexible to numerous mission constraints



- Combine monotonic basin hopping (MBH) & sequential quadratic programming (SQP) → MBH+SQP
- Stochastic, global search scheme
- No initial guess required
- Adept at multi-modal problems w/ clustered local minima
- Stochastic “hops” evaluated from base solution



- MGAnDSMs structured within multi-objective hybrid optimal control algorithm (discrete & continuous variables)
- Multi-objective genetic algorithm (GA) serves as outer loop systems optimizer around direct-method inner loop trajectory optimizer
 - Outer loop: non-dominated Sorting Genetic Algorithm II (NSGA-II) searches over discrete mission parameters, defining trajectory problem for inner loop
 - Variables include: Flyby body, target body, launch vehicle, launch C3, launch epoch
 - Inner loop: MBH+SQP solves trajectory problem & establishes obj. func. values
 - Generates representation of Pareto front (optimal tradeoff between objectives)

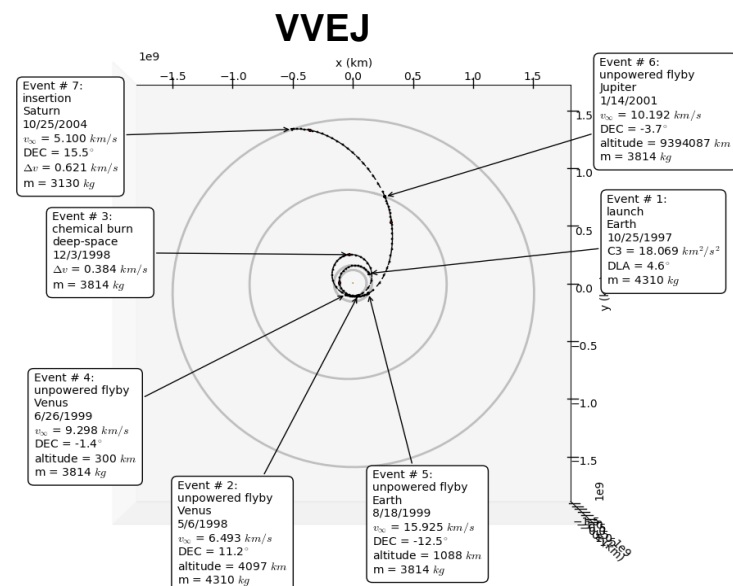


Cassini Test Case Comparison



- Established global optimum serves a test case for comparison of MGAnDSMs & a Lambert-based transcription (MGADSMk); one DSM allowed
- MGAnDSMs identified global optimum in 10 out of 10 cases
- MGADSMk identified solution w/in 50 m/s of global optimum, but not the global optimum (1.004 km/s) after an 8 hour run time
- Order of magnitude improvement in time to best solution
 - Median time is 25x faster

Run Number	MGAnDSMs Time to Identify Global Optimum (minutes)	MGADSMk Time to Identify Best Solution (minutes)	MGADSMk Time Identify Solution within 50 m/s of Global Optimum (minutes)	MGADSMk Best ΔV (km/s)
1	5.3	223.8	220.3	1.025
2	7.9	140.9	32.9	1.043
3	8.8	260.8	149.3	1.026
4	12.8	74.3	74.3	1.014
5	63.1	178.1	122.0	1.020
6	4.3	295.4	112.0	1.034
7	5.0	471.3	465.7	1.039
8	6.3	382.9	328.5	1.049
9	9.4	80.6	58.4	1.033
10	38.4	63.6	63.6	1.038
Mean	16.1	217.2	162.7	1.032
Median	8.4	200.9	117.0	1.033



Example Problem: Multi-objective Optimization to Jupiter



- Evaluated MGA_nDSMs multi-objective capability on a multiple gravity assist to Jupiter with up to 2 DSMs per phase
- Flyby bodies are varied with any combination up to 5 bodies
- Three objectives: maximize $\log_{10}(\text{final mass})$, minimize TOF, minimize Jupiter arrival C3
 - Pareto front: 3D surface of equally optimal solutions
- Population size of 256 and the MBH+SQP inner loop is allowed to run for 40 minutes

Outer-loop Design Variables

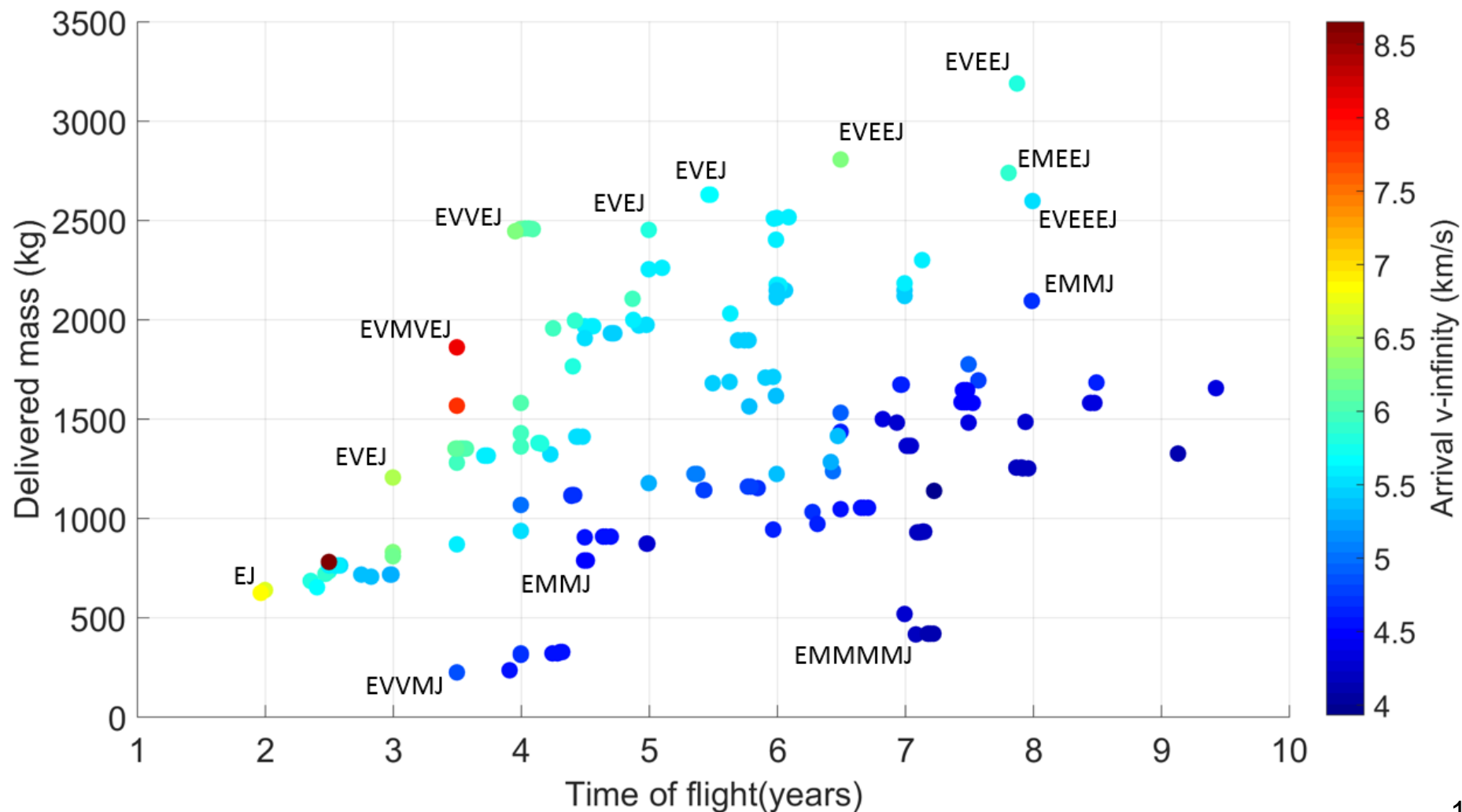
Design Variable	Value	Resolution
Launch window open epoch	{1/1/2021, 1/1/2022, 1/1/2023, 1/1/2024}	1 year
Flyby body	{Venus, Earth, Mars, null, null, null}	n/a
Flight time	[730, 3467.5] days	182.5 days

Common Mission Parameters

Description	Value
Launch window	365.24 days
Launch declination	[-28.5, 28.5] deg
Launch vehicle curve	Atlas V, 551
Chemical I_{sp}	320 s
Jupiter arrival date	Determined by optimizer
Jupiter insertion orbit semi-major axis	10,054,900 km (140.6 R_J)
Jupiter insertion orbit eccentricity	0.911
Maximum number of DSMs	2
Inner-loop objective function	Max: $\log_{10}(\text{final mass})$
Inner-loop run time	40 minutes

Best Non-Dominated Front

- Representation of 3D Pareto front generated after 100 generations
- 3.5 days of run time on 64-core processor

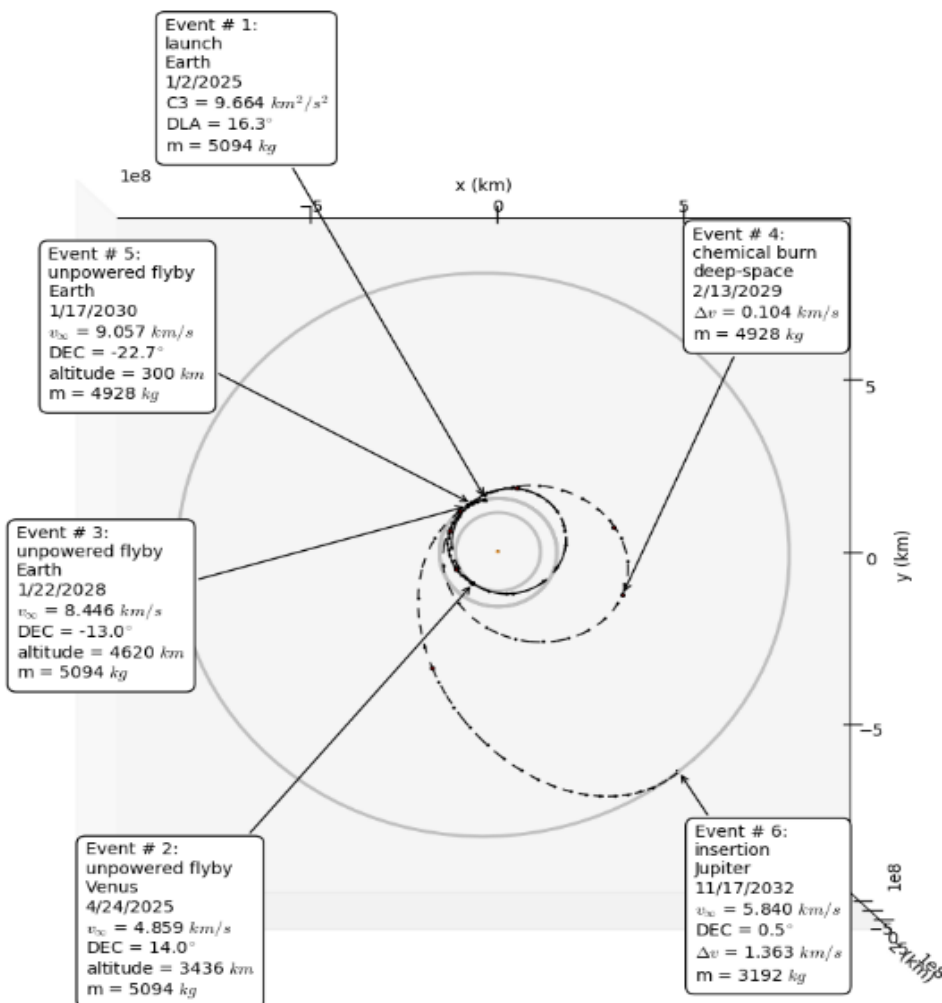


Example Optimal Jupiter Trajectories



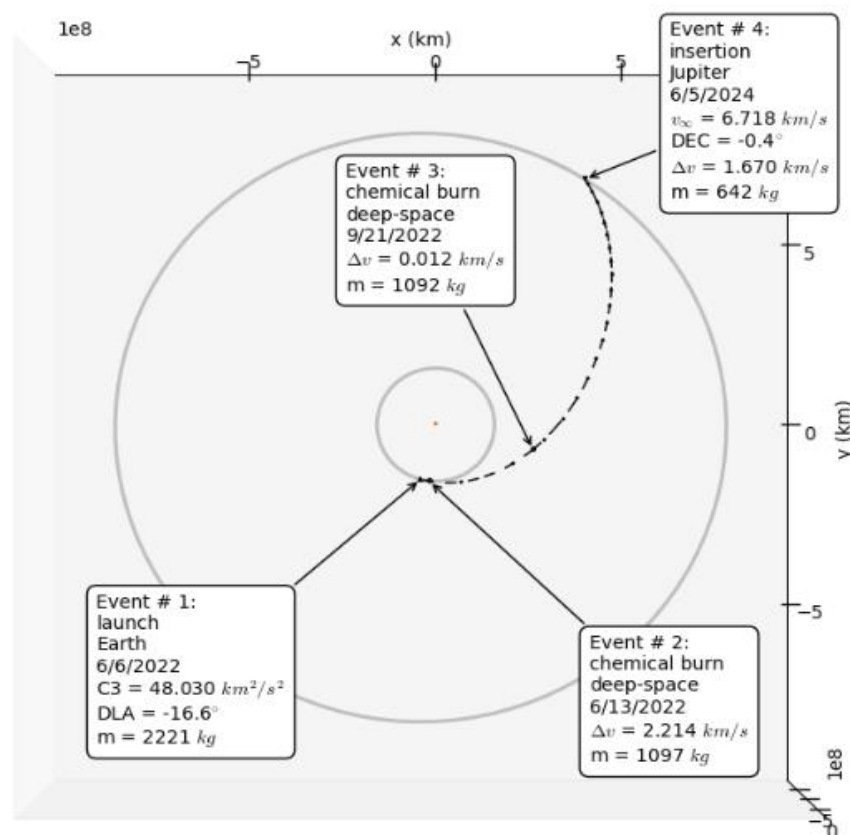
Highest Delivered Mass Trajectory

- EVEEJ sequence
- Delivered mass: 3192 kg
- TOF: 7.9 years
- Arrival C3: 34.1 km/s



Shortest TOF Trajectory

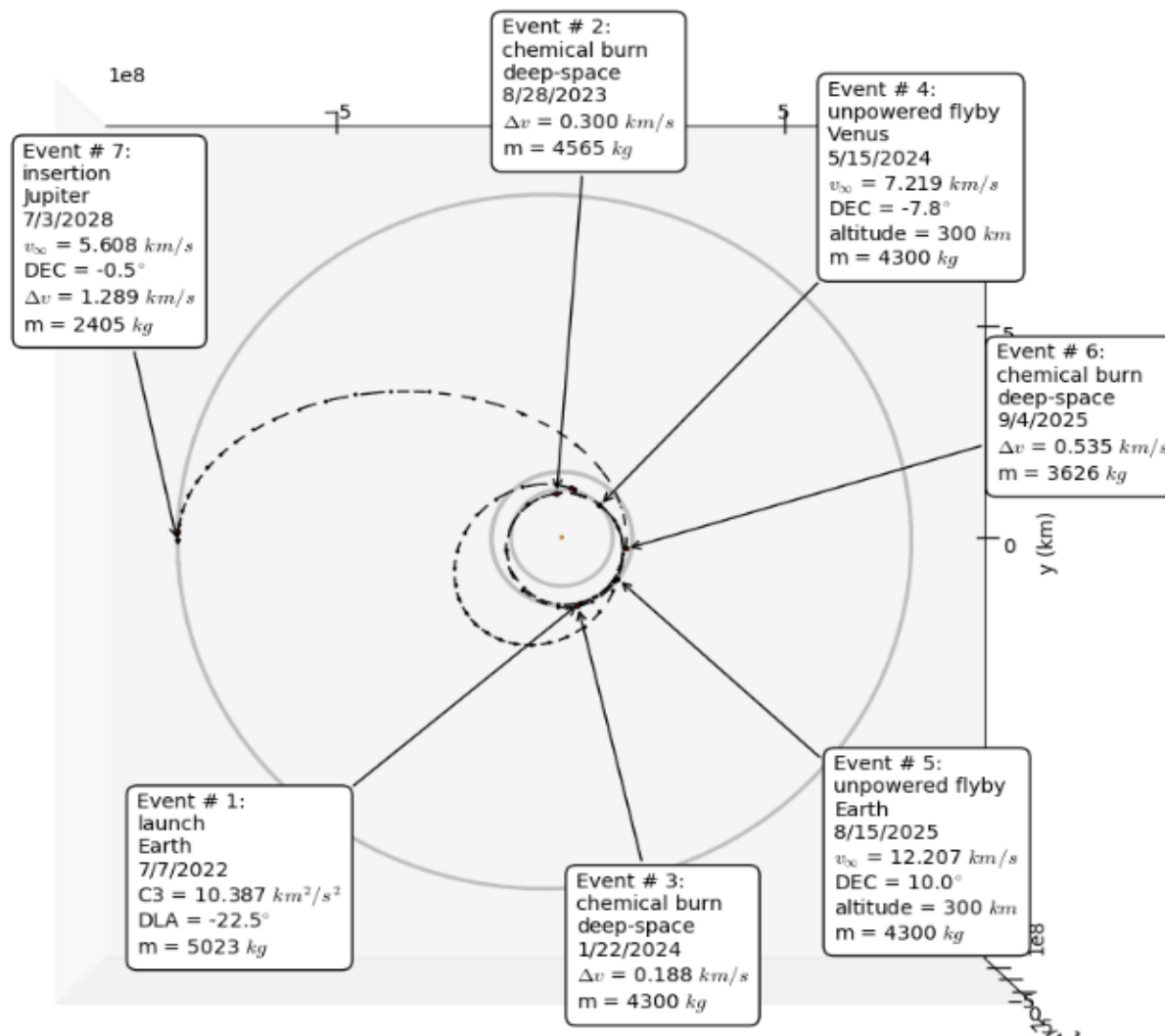
- Direct EJ (2 DSMs optimal)
- Delivered mass: 642 kg
- TOF: 2.0 years
- Arrival C3: 45.1 km/s



Optimal Jupiter Trajectory with 2 DSMs in 1 Phase



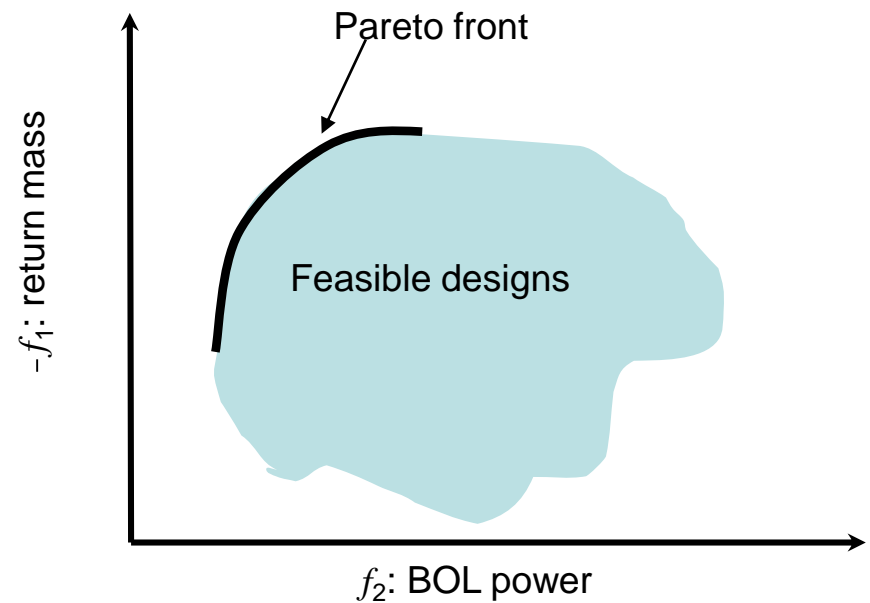
EVEJ sequence



- Developed high-thrust trajectory optimization transcription for any number of maneuvers between flybys
- Multiple shooting framework & analytic derivatives provide robustness and enable outer-loop efficiency
- Formulation allows for an automated, global search without a user-supplied initial guess
- Capability to generate Pareto-optimal solutions using multi-objective hybrid optimal control algorithm
- General applicability to almost any variety of interplanetary mission
 - Flexible to unique trajectory/mission constraints
- Large problems can become computationally tractable

Backup

- Want to optimize any number of mission design metrics
 - e.g., payload mass, TOF, arrival C3
 - Often coupled & competing
 - Fully map mission trade-offs between optimal solutions
- Optimize multiple objectives simultaneously
 - Entire set of optimal solutions
 - Goal: generate representation of Pareto front
 - Traditionally use repetitions of single objective technique



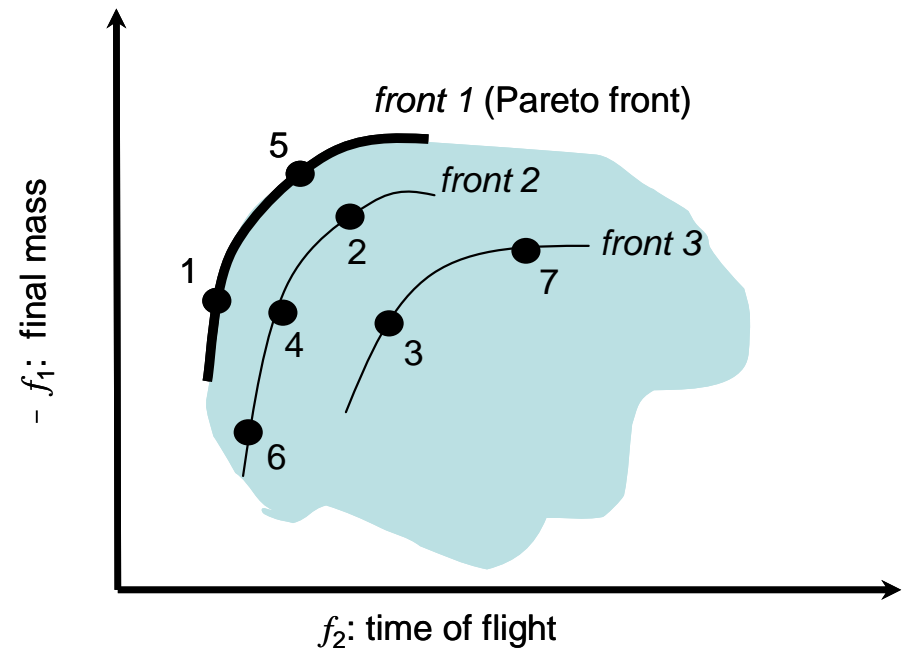
- Develops globally-optimal Pareto solutions using non-dominated sorting
 - Conducts stochastic, global search with population of designs
- Fitness assignment based on “nearness” to Pareto front
 - \mathbf{x}_1 dominates \mathbf{x}_2 if:

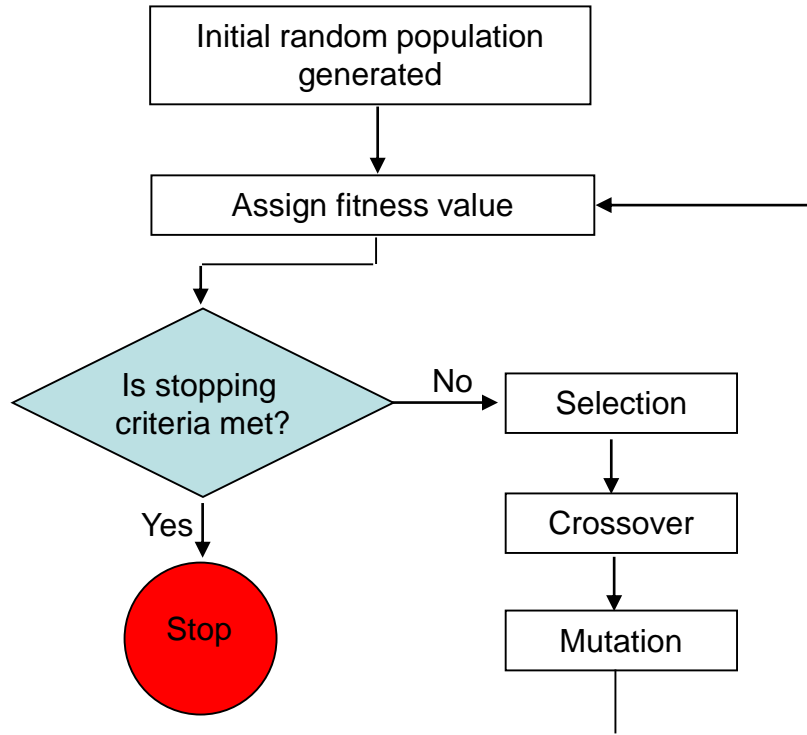
$$\forall p: f_p(\mathbf{x}_1) \leq f_p(\mathbf{x}_2) \quad p = 1, 2, \dots, n_{obj}$$

and

$$\exists p: f_p(\mathbf{x}_1) < f_p(\mathbf{x}_2) \quad p = 1, 2, \dots, n_{obj}$$

- If neither design dominates other, they are non-dominant
- Non-dominated sorting:
 - Assign fitness based on design’s non-dominated front
 - Designs closer to Pareto front get more mating opportunities





- Models Darwinian evolution
 - Mimic natural selection & reproduction
- Searches with population of designs
- Globally search design space
- No initial guess required